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THE ANALYSIS OF STRAINS INDICATED BY MULTIPLE-STRAND

RESISTANCE-TYPE WIRE STRAIN GAGES USED AS ROSETTES

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ADVANCE RESTRICTED REPORT

THE ANALYSIS OF STRAINS INDICATED BY MULTIPLE-STRAND RESISTANCE-TYPE WIRE STRAIN GAGES USED AS ROSETTES

By Morris F. Dow

SUMMARY

Methods are given for making the necessary corrections to the strains indicated by multiple-strand. resistance-type wire strain gages used singly or as rosettes to measure strains at an angle to the principal strain. The results of tests to determine the validity of the methods of correction are reported.

A rosette arrangement having several advantages over the arrangements now commonly in use is described.

INTRODUCTION

The resistance—type wire strain gages are particularly adaptable for use in strain rosettes because of compactness and facility of reading. When used as rosettes, three of the gages are generally arranged at angles of 45° or 60° to each other or, less frequently, a fourth gage is added to the 45° arrangement. In any arrangement, only one gage in the rosette measures the strain at the gage length in question and the other gages are mounted above or to the side of that gage.

A strain-rosette computing machine, described in reference 1, has been developed for calculating principal strains from the individual strain-gage readings of a three-gage 60° rosette or a four-gage 45° rosette. This machine cannot be conveniently used with a three-gage 45° rosette. A rosette arrangement that combines the features of the 45° and 60° rosettes and is particularly adapted for use with this machine is herein described.

It has been recognized that rosettes made up of resistance—type wire strain gages so wound that some of

the resistance wire is at an angle to the axis of the gage are affected by the component of strain acting at that angle. Strains having an angular component different from the angular component present in the calibration of the gage will accordingly cause errors in strain indication that are a function of the geometry of the grid of the resistance wire. A method by Baldwin Southwark for correcting these errors, by making a correction to each gage reading, for the commercial SR-4 type R-1 rosette is given in appendix A.

Because the magnitude and direction of the principal strains are usually of greater interest than individual gage readings, a correction to be directly applied to the indicated principal strains is generally more convenient than corrections to each strain indication. In the use of a rosette computing machine, some time might also be saved by making corrections to the principal strains rather than to the original gage readings. Methods, adaptable to any wire strain gage, for making these corrections are described in this report, together with the results of tests of SR-4 rosettes to determine the accuracy of the corrections.

T-DELTA ROSETTE

The strain rosette arrangements now in common use, the three—and four—gage 45° star rosettes and the three—gage 60° delta rosettes, have advantages and disadvantages peculiar to each arrangement. A rosette made up of gages at 60° forming a delta, with a fourth gage in the center of the delta perpendicular to one of the three other gages, designated T—delta appears to offer a maximum of the advantages of the usual arrangements. These advantages are:

- l. The two perpendicular arms may be mounted in the expected directions of the principal strains, which minimizes the errors introduced by incorrect angular alinement of the gages.
- 2. The gage at the center may be mounted on the point at which it is desired to measure the strain and along the expected direction of principal strain, and the other three gages will fall approximately as far from that point as the three gages in the usual delta.
- 3. The principal strains may be computed from the rosette even if one of the gages fails during the test.

4. A check on the accuracy of measurement is provided by the fourth gage.

CORRECTION OF STRAINS INDICATED BY SINGLE

MULTIPLE-STRAND GAGE

The strain indications from a multiple-strand resistance-type wire gage at an angle 0 to the direction of a simple tensile or compressive stress (when one of the principal stresses is zero) may be corrected for the effect of the width of the gage as follows:

l. The gage factor \mathbf{G}_0 for the gage mounted in the direction of a simple tensile or compressive stress is determined by

$$G_{o} = \frac{\Delta R}{R} \tag{1}$$

in which

R resistance of gage

AR change in resistance

eo strain along stress direction

2. A correction factor Q is calculated from the equation

$$Q = \frac{1}{1 - (1 + \mu) \frac{W}{L}} \left\{ 1 - \left[\frac{2(1 + \mu) \cos 2\theta}{(1 - \mu) + (1 + \mu) \cos 2\theta} \right] \frac{W}{L} \right\}$$
 (2)

in which

μ Poisson's ratio for material used

L total length of resistance wire in gage

W width of grid of resistance wire

The derivation of equation (2) is given in appendix B.

3. The strain at the angle 0 may now be calculated as

$$\mathbf{e}_{\dot{\boldsymbol{\theta}}} = \frac{\frac{\Delta \mathbf{R}}{\mathbf{R}}}{\mathbf{G}_{\boldsymbol{\theta}}} \qquad (3)$$

in which $G_A = G_0Q$.

Values of G_{θ} for an SR-4 type A-1 gage for which $G_{0}=2.09$, L = 5.01 inches, and W = 0.16 inch are plotted in figure 1.

CALCULATION OF TRUE PRINCIPAL STRAINS FROM

INDIVIDUAL GAGE READINGS

As shown by Hill in reference 2 and Sibert in reference 5, the true principal strains may be calculated from the true strains measured on three or nore intersecting gage lines from the formulas

$$e_{X} = K(Y + 1) \tag{4}$$

$$\mathbf{e}_{\mathbf{y}} = \mathbb{K}(\mathbf{Y} - \mathbf{1}) \tag{5}$$

in which $\mathbf{e_X}$ and $\mathbf{e_y}$ are the true principal strains. The constants K and Y are determined from the arrangement of the rosette and strain readings. For a rosette composed of three gages nounted on gage lines intersecting at angles of 45°

$$\mathbb{I}_{45} = 0.707 \sqrt{\left(e_{1_{45}} - e_{2_{45}}\right)^2 + \left(e_{3_{45}} - e_{3_{45}}\right)^2}$$
 (6)

$$Y_{45} = \frac{^{6}1_{45}}{^{2}K_{45}} + {^{6}3}_{45} \tag{7}$$

in which elas, e245, and e345 are the true strains along the gage lines. Throughout this report true strains

will be designated e. with appropriate subscripts. Strains indicated by wire gages will be designated e. For a rosette composed of gages mounted at angles of 60°

$$X_{60} = 0.471 \sqrt{\left(e_{1_{60}} - e_{2_{60}}\right)^{8} + \left(e_{1_{60}} - e_{3_{60}}\right)^{8} + \left(e_{2_{60}} - e_{3_{60}}\right)^{8}} + \left(e_{2_{60}} - e_{3_{60}}\right)^{8}}$$

$$Y_{60} = \frac{e_{1_{60}} + e_{2_{60}} + e_{3_{60}}}{3K_{60}}$$
(9)

For a rosette composed of a delta plus a fourth gage at the center, that is, for a T-delta rosette,

$$K_{60T} = 0.577 \sqrt{\left(e_{1_{60T}} - e_{2_{60T}}\right)^8 + 3\left(e_{2_{60T}} - e_{4_{60T}}\right)^8}$$
 (10)

or

$$\mathbf{x}_{60T} = 0.108 \sqrt{\left(\mathbf{e}_{1_{60T}}^{-\mathbf{e}_{2_{60T}}}\right)^{2} + \left(\mathbf{e}_{1_{60T}}^{-\mathbf{e}_{3_{60T}}}\right)^{2} + 3\left[\left(\mathbf{e}_{2_{60T}}^{-\mathbf{e}_{1_{60T}}}\right)^{2} + \left(\mathbf{e}_{3_{60T}}^{-\mathbf{e}_{1_{60T}}}\right)^{2}\right]} (10a)$$

$$\mathbf{x}_{60T} = \frac{\mathbf{e}_{1_{60T}}^{-\mathbf{e}_{3_{60T}}} + \mathbf{e}_{4_{60T}}^{-\mathbf{e}_{1_{60T}}}}{2K_{60T}}$$
(11)

in which e_4 is the center gage, perpendicular to e_1 .

The true principal strains may be calculated from the strains indicated by multiple-strand resistance-type wire gages by use of these same formulas if the following method is used:

1. The true strain-sensitivity factor S for the resistance wire in the gage is used for obtaining strain readings. The true strain-sensitivity factor is equal to the gage factor for a single-strand wire gage and may be determined experimentally or calculated as

$$S = G_0Q_{45}$$
 : (12)

in which Q_{45} is given by equation (2) for $\theta = 45^{\circ}$.

- 2. The constants K' and Y' are calculated from equations (6) to (11) by substituting e' for e throughout.
 - 5. These constants are corrected as

$$K_{r} = K! \left(\frac{L}{L - 2W} \right) \tag{13}$$

$$Y_{r} = Y' \left(\frac{L - 2W}{L} \right) \tag{14}$$

Then

$$e_{x} = K_{r}(Y_{r} + 1) \qquad (15)$$

$$\mathbf{e}_{\mathbf{y}} = \mathbb{X}_{\mathbf{r}} (\mathbf{Y}_{\mathbf{r}} - 1) \tag{16}$$

CALCULATION OF TRUE PRINCIPAL STRAIMS FROM

INDICATED PRINCIPAL STRAINS

The principal strains e_x and e_y , calculated directly from the strain indications of the multiplestrand wire rosettes as by a rosette computing machine, may also be corrected as follows:

- 1. The true strain-sensitivity factor S is used for calculating all gage readings.
- 2. The principal strains are directly calculated from these readings.
 - 3. A constant C! is determined as

$$C_{45}^{!} = \frac{e_1^{!} + e_3^{!}}{2}$$
 for a 45° rosette (17)

$$C_{60}' = \frac{e_1' + e_2' + e_3'}{3}$$
 for a 60° rosette (18)

$$C_{60T}^{i} = \frac{e_1^{i} + e_4^{i}}{2}$$
 for a T-delta rosette (19)

Or

$$C' = \frac{e_{x}' + e_{y}'}{2} \text{ for any rosette}$$
 (20)

in which $e_x^{}$ and $e_y^{}$ are the indicated principal strains.

- 4. This constant C' is deducted from the major principal strain $e_{\mathbf{x}}$ calculated from the strain indications.
- 5. The remainder is multiplied by the factor $\left(\frac{L}{L-2W}\right)$, which gives $K_{\mathbf{r}}$. Then

$$\mathbf{e}_{\mathbf{x}} = \mathbf{C}^{\dagger} + \mathbf{K}_{\mathbf{r}} \tag{21}$$

$$\mathbf{e}_{\mathbf{y}} = \mathbf{C}_{\mathbf{i}} - \mathbf{K}_{\mathbf{r}} \tag{33}$$

EXTENSION OF HILL'S SEMIGRAPHICAL METHOD TO ANALYSIS

OF MULTIPLE-STRAND WIRE ROSETTES

Hill's semigraphical method may be readily extended for use with multiple-strand gages. Many of the following steps are taken directly from reference 2.

- l. The true strain—sensitivity factor for the wire in the gage is determined. The true factor may be experimentally determined or calculated by multiplying the gage factor for the gage as normally calibrated by the value of Q from equation (1) for $\theta=45^\circ$.
- 2. A master curve of the function $f=\cos 2\theta$ is constructed on transparent paper or cloth, in which f is plotted against θ for the region $-45^{\circ}<\theta<135^{\circ}$. This master curve may be used each time that the method is applied to a set of strain data.
- 3. The unit strains e_1^i , e_2^i , e_3^i indicated on the three gages when the true strain sensitivity is used, are

divided by an adjustment factor K' to be determined from the appropriate equation (6), (8), or (10), as

the appropriate equation (6), (8), or (10), as
$$K_{60T}' = 0.577 / \left(e_{1_{60T}}' - e_{2_{60T}}'\right)^{8} + 3 \left(e_{2_{60T}}' - e_{4_{60T}}'\right)^{8}$$
4. The adjusted strain readings are plotted as ordinated at the strain readings are plotted as ordinated at the strain readings.

- 4. The adjusted strain readings are plotted as ordinates to the same scale as used for f-values on the master curvo. The 0-values, that is, angles between gage lines, are the abscissas. The choice of origin for this graph is immaterial; the relative positions of the three gage lines are important.
- 5. An auxiliary horizontal line representing the average of the adjusted strain values for gage lines 1 and 3 is drawn. The ordinate Y' for this line may be found from equation (7), (9), or (11), as

$$Y' = \frac{e_{160T}'}{Z'} + \frac{e_{460T}'}{K'}$$
 (11)

6. A new adjustment factor $X_{\mathbf{r}}$ is determined from the equation

$$K_{\mathbf{r}} = K! \left(\frac{L}{L - 2W} \right) \tag{13}$$

7. A second auxiliary horizontal line is drawn, for which the ordinate Yc from the origin of the plotted strains is given by

$$\lambda^{G} = \lambda_{i} \left(\frac{T}{SM_{i}} \right) \tag{.32}$$

- 8. The master curve is placed over the graph, with its horizontal axis coinciding with the first auxiliary line of the graph, and in such a position that the plotted points fall on the master curve.
- 9. The abscissas of the maximum and the minimum points of the master curve, referred to the origin of the graph, indicate the directions of the principal strains and the

ordinates, referred to the second auxiliary line, indicate the magnitude of their adjusted values. In order to obtain the true magnitude of the principal strains, these values must be multiplied by the adjustment factor Kr.

10. The principal stresses may be found from the principal strains by the formulas

$$\sigma_{\max} = \frac{\mathbb{E}\left(e_{\max} + \mu e_{\min}\right)}{1 - \mu^{2}} \tag{24}$$

$$\sigma_{\min} = \frac{\mathbb{E}\left(e_{\min} + \mu e_{\max}\right)}{1 - \mu^2} \tag{25}$$

in which E = Young's modulus.

11. The directions of the principal stresses may be found from the graph or may be calculated from

$$\left(\cos 2\theta\right)_{60T} = \frac{e_{1_{60T}}! - e_{4_{60T}}!}{2K_{60T}!} \tag{26}$$

12. The maximum shear stress τ_{max} , acting at 45° to the principal stresses, may be found from

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} \tag{27}$$

CHECK OF ACCURACY OF CORRECTIONS

Because the stresses at the center of a polygonal dural block of 16 sides loaded in compression between parallel pairs of its 16 sides are combined, such a specimen was used to compare the strains measured by Tuckerman optical strain gages and by SR-4 type R-1 wire strain rosettes. A photograph of the polygonal specimen is given as figure 2. This specimen had the further advantage that the principal strains could be inclined at angles of $22\frac{10}{10}$, 45° , $67\frac{1}{2}^{\circ}$, and 90° to the axis of any gage in the rosette.

The properties of the specimen were first surveyed by mounting Tuckerman gages on the front and the rear of the block at the center of the block and parallel to the line drawn between the midpoints of two opposite sides. The specimen was loaded in a hydraulic testing machine accurate to one—half of 1 percent, at angles of 0°, 22½°, 45°, 67½°, and 90° to the axis of the Tuckerman gages and the strains measured at each angle for load increments of 100,000 pounds. The gages were then rotated 45° and the survey repeated; surveys were made at each 45° rotation of the gages.

The properties of the specimen were found to be sufficiently uniform. An average of the survey could therefore be used to determine the strains at any angle at the center of the block for a load increment of 100,000 pounds. The maximum variation from the average properties as indicated by the Tuckerman gages was 0.91 percent, or a strain of 0.000009 inch per inch.

The Tuckerman gages were replaced by SR-4 rosettes and the surveys repeated. The results are given in table 1 for the average true strains, for the average electrical readings uncorrected, corrected by the Baldwin Southwark method, and corrected by the methods of the present report.

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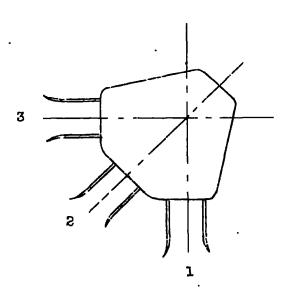
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APPENDIX A

BALDWIN SOUTHWARK METHOD OF COMPUTING STRAINS

FROM TYPE H-1 ROSETTE

The following sketch and method of computing strains from an SR-4 type R-1 rosette was obtained from directions accompanying the gages, which were manufactured by Baldwin Southwark Division of The Baldwin Locomotive Works.



The numbers 1, 2, and 3 indicate the gage axes. The true strains along the gage axes are designated e₁, e₂, e₃. The strains indicated by instrument are designated e₁, e₂, and e₃. Then, according to Baldwin Southwark

$$e_1 = e_1' - \frac{1}{45} e_3'$$
 $e_2 = 1.02e_2' - \frac{1}{45} (e_1' + e_3')$
 $e_3 = e_3' - \frac{1}{45} e_1'$

The symbols e₁, e₂, and e₃ have here been used instead of R₁, R₂, and R₃, given in the original directions, to conform with the other symbols in the present report.

APPENDIX B

ANALYSIS OF STRAIN INDICATIONS GIVEN BY

MULTIPLE-STRAND WIRE GAGES

The true strain e_{θ} at an angle θ to the direction of principal strain is given by the equation from reference 1

$$\mathbf{e}_{\theta} = \frac{1}{2} \left[(\mathbf{e}_{\mathbf{x}} + \mathbf{e}_{\mathbf{y}}) + (\mathbf{e}_{\mathbf{x}} - \mathbf{e}_{\mathbf{y}}) \cos 2\theta \right] \qquad (A-1)$$

in which e_x and e_y are the principal strains.

If the gage is attached at an angle θ to the principal strain, the length of wire in the gage minus the width of the wire grid is subjected to the strain e_{θ} and the width of the gage is subjected to the strain at the angle (θ + 90°). The strain indicated by the gage is then expressed as follows:

$$\frac{\Delta R_{\theta}}{R_{\theta}} = c \left\{ \frac{1}{2} \left[(e_{x} + e_{y}) + (e_{x} - e_{y}) \cos 2\theta \right] \left(\frac{L - W}{L} \right) + \frac{1}{2} \left[(e_{x} + e_{y}) + (e_{x} - e_{y}) \cos 2(\theta + 90^{\circ}) \right] \left(\frac{W}{L} \right) \right\} (A-2)$$

in which

c constant

 $\frac{\Delta R}{R}$ change in resistance of gage divided by gage resistance

 e_x and e_y principal strains

L and W previously defined .

The strain indicated by the gage as normally calibrated; that is, with the axis of the gage parallel to the direction of simple tension or compression is therefore

$$\frac{\Delta R_c}{R_0} = c \left\{ \frac{1}{2} \left[(\mathbf{e_x} - \mu \mathbf{e_x}) + (\mathbf{e_x} + \mu \mathbf{e_x}) \right] \right. \\ \left. \left(\frac{L - W}{L} \right) + \left. \frac{1}{2} \right. \left[(\mathbf{e_x} - \mu \mathbf{e_x}) - (\mathbf{e_x} + \mu \mathbf{e_x}) \right] \left(\frac{W}{L} \right) \right\}$$

or

$$\frac{\Delta R_0}{R_0} = c \left[e_x \left(\frac{L - W}{L} \right) - \mu e_x \left(\frac{W}{L} \right) \right] \qquad (A-3)$$

For a simple tensile or compressive strain of unity $(e_X=1,\ e_Y=-\mu)$, the result Q of dividing the strain indicated by the gage mounted at an angle θ to the direction of e_X , and so subjected to a strain e_θ , by the strain indication that would be calculated at that angle from substituting e_θ in equation (A-3) is

$$Q = \frac{\frac{\Delta R_{\theta}}{\Delta R_{\theta_{0}}}}{\frac{R_{\theta_{0}}}{R_{\theta_{0}}}} = \frac{\frac{1}{2} \left[(1-\mu) + (1+\mu)\cos 2\theta \right] \left(\frac{L-W}{L} \right) + \frac{1}{2} \left[(1-\mu) + (1+\mu)\cos 2(\theta+90^{\circ}) \right] \left(\frac{W}{L} \right)}{\frac{1}{2} \left(\frac{L-W}{L} - \mu \frac{W}{L} \right) \left[(1-\mu) + (1+\mu)\cos 2\theta \right]}$$

or
$$Q = \frac{[(1-\mu)+(1+\mu)\cos 2\theta](L-W) + [(1-\mu)+(1+\mu)\cos 2(\theta+90^{\circ})](W)}{(L-W-\mu W)[(1-\mu)+(1+\mu)\cos 2\theta]}$$

or
$$Q = \frac{1}{1-(1+\mu)\frac{W}{L}} \left\{ 1 - \left[\frac{2(1+\mu)\cos 2\theta}{(1-\mu)+(1+\mu)\cos 2\theta} \right] \frac{W}{L} \right\}$$
 (A-4)

In order to show that Hill's semigraphical analysis may be applied to the readings given by multiple-strand wire gages to give the true principal strains, it is necessary to show that the variation in indicated strain with change of angle is a function of the form

If equation (A-2) is rewritten

$$\frac{\Delta R_{\theta}}{R_{\theta}} = (A + B \cos 2\theta) \frac{L - W}{L} + (A - B \cos 2\theta) \frac{W}{L} \qquad (A-5)$$
in which
$$A = \frac{e_{x} + e_{y}}{2}$$

$$B = \frac{e_x - e_y}{2} \cos 2(\theta + 90^\circ) = -\cos 2\theta \left(\frac{e_x - e_y}{2}\right)$$

Then

and

$$\frac{\Delta E_{\theta}}{\Delta C} = A + B \frac{(L - 2W)}{L} \cos 2\theta \qquad (\Delta - 6)$$

Equation (A-6) is in the required form and Hill's semigraphical method may be used if the proper corrections are applied to the adjustment factor I and to the ordinate of the auxiliary horizontal line. The constant is given in reference 2 by

$$K = 0.707 \sqrt{(e_1 - e_2)^2 + (e_2 - e_3)^2}$$

in which e1, e2, and e3 are the true strains measured on gage lines intersecting at angles of 45°.

In order to calculate the new adjustment factor it is necessary only to find the relation between the principal strain as indicated by multiple-strand gages and the true principal strains. The principal strains indicated by the multiple-strand gage, when the true strainsersitivity factor is used, are

$$e_{x}' = \frac{e_{1}' + e_{3}'}{2} + K'$$
 (A-7)

$$e^{\lambda_i} = \frac{5}{e^{1_i + e^{2_i}}} - K_i$$
 (7-8)

with

$$K_{45}^{1} = 0.707 \sqrt{(e_{1}^{1} - e_{2}^{1})^{2} + (e_{2}^{1} - e_{3}^{1})^{2}}$$

in which e_1' , e_2' , and e_3' are the strains indicated by the wire gages.

Equation (A-2) also gives ex¹ and ey¹ when $\theta = 0^{\circ}$ and $\theta = 90^{\circ}$, respectively, as follows:

$$e_{x}^{1} = \frac{1}{2} [(e_{x} + e_{y}) + (e_{x} - e_{y})] (\frac{L - W}{L}) + \frac{1}{2} [(e_{x} + e_{y}) - (e_{x} - e_{y})] (\frac{W}{L})$$

$$\mathbf{e}_{\mathbf{x}^{\dagger}} = \mathbf{e}_{\mathbf{x}} \left(\frac{\mathbf{L} - \mathbf{W}}{\mathbf{L}} \right) + \mathbf{e}_{\mathbf{y}} \left(\frac{\mathbf{W}}{\mathbf{L}} \right) \tag{A-9}$$

and

$$e_{\mathbf{y}}^{t} = e_{\mathbf{y}} \left(\frac{\mathbf{L} - \mathbf{W}}{\mathbf{L}} \right) + e_{\mathbf{x}} \left(\frac{\mathbf{W}}{\mathbf{L}} \right)$$
 (A-10)

Combining equations (A-7) and (A-9) and equations (A-8) and (A-10) and solving for $e_{\bf x}$ and $e_{\bf y}$ gives

$$\mathbf{e}_{\mathbf{x}} = \frac{\mathbf{e}_{1} \cdot + \mathbf{e}_{3} \cdot}{2} + \mathbb{K} \cdot \left(\frac{\mathbf{L} - 2\mathbf{W}}{\mathbf{L}} \right) \tag{A-11}$$

$$e_y = \frac{e_1' + e_3'}{2} - K' \left(\frac{L}{L - 2W}\right)$$
 (A-12)

The expression for K, as given by Hill, may be written

$$X = 0.707 \sqrt{(e_X - e_{45})^2 + (e_{45} - e_y)^2}$$
 (A-13)

in which e_{45} is the true strain at an angle or 45° to principal strains. From equation (A-2)

$$e_{45} = \frac{1}{2} (e_x + e_y) \left(\frac{L - W}{L} \right) + \frac{1}{2} (e_x + e_y) \left(\frac{W}{L} \right)$$
 (A-14)

Combining equations (A-11), (A-12), and (A-14) gives

$$e_{145} = \frac{1}{2} \left[\frac{e_1' + e_3'}{2} + K' \left(\frac{L}{L-2W} \right) + \frac{e_1' + e_3'}{2} - K' \left(\frac{L}{L-2W} \right) \right] \left(\frac{L-W}{L} + \frac{W}{L} \right) (A-15)$$
and.
$$e_{45} = \frac{e_1' + e_3'}{2} \qquad (A-16)$$

From the expressions for $e_{\rm X},~e_{\rm 45},~$ and $e_{\rm y},~$ an expression for $K_{\rm r}$ may be written

$$K_r = 0.707 \sqrt{(e_x - e_{45})^2 + (e_{45} - e_y)^2}$$
 (A-13)

$$K_{r} = 0.707 \sqrt{\left[\frac{e_{1}^{1}+e_{3}^{1}}{2} + K^{1}\left(\frac{L}{L-2W}\right) - \frac{e_{1}^{1}+e_{3}^{1}}{2}\right]^{2} + \left[\frac{e_{1}^{1}+e_{3}^{1}}{2} - \frac{e_{1}^{1}+e_{3}^{1}}{2} + K^{1}\left(\frac{L}{L-2W}\right)\right]^{2}} (A-17)$$

$$K_r = K! \left(\frac{L}{L - 2W} \right) \tag{A-18}$$

and for the new ordinate Yr

$$Y_{r} = \frac{\frac{e_{x}}{K_{r}} + \frac{e_{y}}{K_{r}}}{2} \qquad (A-19)$$

$$Y_{r} = \frac{\frac{e_{1}! + e_{3}!}{2} + K_{r} + \frac{e_{1}! + e_{3}!}{2} - K_{r}}{2K_{r}}$$

$$Y_r = \frac{\frac{e_1!}{K_r} + \frac{e_3!}{K_r}}{2}$$
 (:A-20)

or

$$Y_{r} = Y^{t} \left(\frac{L - 2W}{L} \right) \tag{A-21}$$

TABLE 1

COMPARISON OF STRAIN MEASUREMENTS TAKEN WITH TUCKERMAN OPTICAL STRAIN

GAGES AND WITH SR-4 TYPE R-1 ROSETTES

[All strains given in microin. per in.]

Angle (deg)	True	Electrical strain indications				Electrical strains			
	strain	Uncorrected				Corrected as indi-		Corrected by Hill's	
			tor = 2.09) Difference from true value				age readings Difference from true value		Difference from true value
0	-981	- 979	2	-992	-11	-979	 2	- 978	3
11 2	-922	- 919	3	-930	i - g	-91g	ĵţ	-919	3
22녉	-751	-751	0	-758	-7	- 7 ¹ 49	2	-749	2
33 1	-496	- 500	-Jt	- 502	– 6	-74974	2	-74674	2
45	-194	-203	-9	-198	-4	- 193	1	-192	. 2
56]	121	108	-13	119	-2	. 124	3	122	1
67출	362	345	-17	362	0	362	o	362	0
78 3	534	515	-19	535	1	534	0	534	0
90	593 دو	572	-21 \	594	1	593	0	593	0

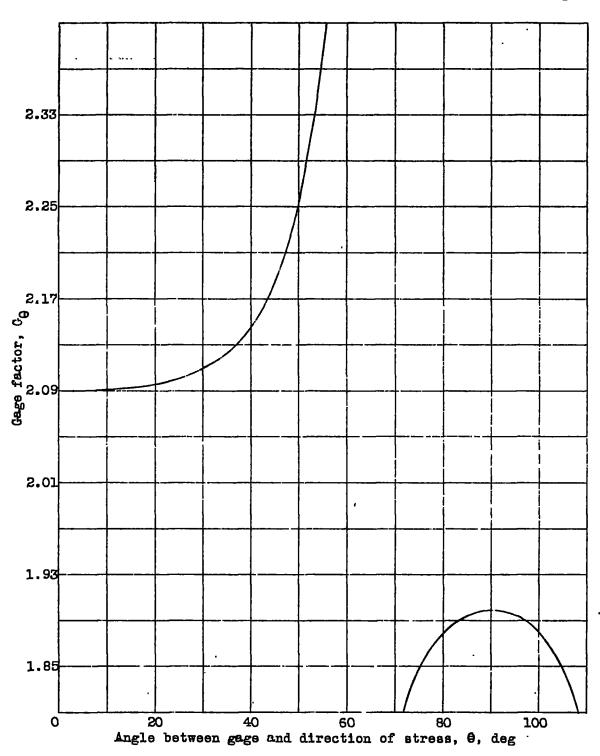


Figure 1.- Variation in gage factor with angle between SR-4 type A-1 gage and direction of stress.G₀ =2.09; L=5.01 inches; W=0.16 inch.

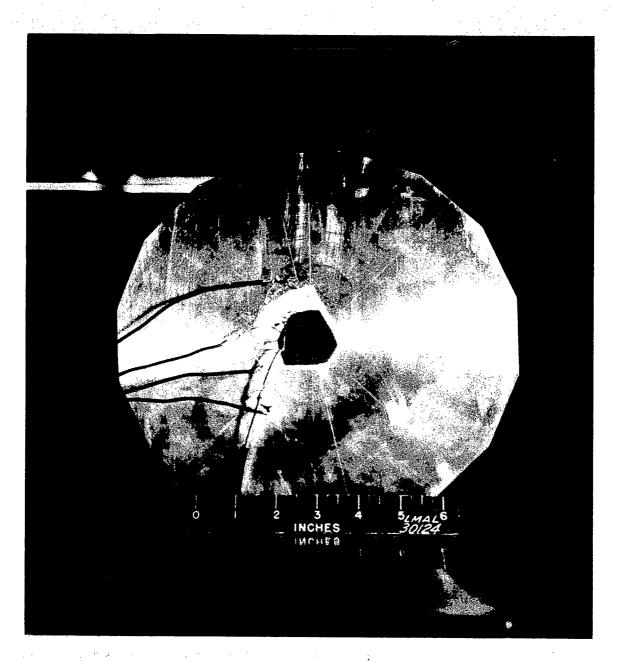


Figure 2. Polygonal specimen for testing rosettes.

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3 1176 01365 5494